GESTURES AS FACILITATORS TO PROFICIENT MENTAL MODELERS

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Gestures are profoundly integrated into our communication. This study focuses on the impact that gestures have in a mathematical setting, specifically in an undergraduate calculus workshop. There was strong correlation between diagramming and the two types of gestures identified in this study (i.e., dynamic and static gestures). Dynamic and static gestures were part of the students' constructive thinking, whether it was related to the manner in which they viewed the problem or the construction of their diagrams. Nonetheless, gestures played a strong role in the students' problem solving and the manner in which the gestures were utilized provided insight into their constructive thinking.

Keywords: Classroom Discourse; Instructional Activities and Practices; Modeling; Problem Solving

Background

There have been numerous studies conducted on gestures and their presence in the educational environment especially in math and science related fields (Rasmussen, Stephan, & Allen, 2004; Chu & Kita, 2011; Goldin-Meadow, Cook, & Mitchell, 2009; Scherr, 2008). For example, Rasmussen, Stephan, and Allen (2004), studied gesturing in a differential equations class where they observed mathematical classroom practices become what they call taken as shared (TAS) ideas for the participants. Through their theoretical perspective they formed a gesture/argumentation dyad, which they used to analyze the gesturing that occurred among the classroom community. In short, their analysis was predicated upon students and teachers as opposed to just students.

Goldin-Meadow, Cook, and Mitchell (2009) explored the impact of teaching third and fourth graders how to gesture while solving a specific type of problem such as 3+4+6=__+6. In their study the participants were separated into three groups: the first was taught a correct gesture, the second group was taught a partially correct gesture, and the third group was not told to gesture at all. Although Goldin-Meadow, Cook, and Mitchell (2009) based their research on the manipulation of gesturing during a math lesson, their findings shed light on the idea that gestures act as an aid when it comes to problem solving for the children who gestured correctly or partially correctly as opposed to those who were not taught to gesture during the lesson.

Studies conducted by Engelke (2004, 2007), were based on understanding students' thought processes in related rates problems. She found that many students fail to understand these types of problems because there is a lack of transformational/covariational reasoning, which pinpoints students' deficiencies in geometry as well as being able to apply mathematical concepts to problems (i.e. similar triangles, substitution, and function composition). As revealed in Engelke (2007) function composition is a necessary tool when dealing with related rates problems. Engelke, Oehrtman, and Carlson's (2005) study highlighted the fact that not much research has been conducted in regard to student understanding of function composition. Other studies indicate that students tend to develop their notion of functions throughout their undergraduate years (Carlson, 1998).

Visualization and representation are dynamic in nature and are an important part of being able to solve word problems (Booth & Thomas, 2000; Carlson & Bloom, 2005; Cifarelli, 1998; Gravemeijer, 1997; Johnson-Laird, 1983; Lucangeli, Tressoldi, & Cendron, 1998; Simon, 1996). Johnson-Laird (1983) extensively discussed three types of mental representation of which two we considered "mental models which are structural analogues of the world, and images which are the perceptual correlates of models from a particular point of view" (p. 165). It has been theorized that in order to understand word problems, students benefit from drawing a diagram and trying to understand the relationships their diagrams represent of the given situation (Engelke, 2007). Visual and analytical skills are essential for students to understand mathematical concepts and construct mental models (Haciomeroglu, Aspinwall, & Presmeg,

2010). Hegarty and Kozhevnikov (1999) described visual-spatial representations as schematic or pictorial. The latter obstructs students from fully understanding the mathematics behind the problem, while the former encourages students to think about the problem in a more abstract manner. Gestures are a form of visual-spatial representation and we investigate how such representations facilitate the problem solving process. We seek to answer the question: How do students' gestures facilitate contextual problem solving in calculus? Through observing students' use of gestures while solving related rates and optimization problems, we will better understand the mental models and diagrams being created during the problem solving process.

Methods

In this study, we used open and axial open coding to observe three different supplemental instruction (SI) workshops, which consisted of undergraduate students taking first semester calculus. Pseudonyms were given to the participants for privacy purposes. The workshops were led by peer instructors. We focused on related rates problems as well as some optimization problems. We watched the videos specifically attending to students making hand gestures and the diagrams they drew during the problem solving process. The observed groups usually consisted of three students. The groups were given specific problems to complete as a team, although some students worked individually and then shared their ideas with their group members. Students often asked SI leaders for help. We took into account student-to-student and student-to-SI leader interactions.

Results

We identify diagramming as a visual tool, including drawing a picture, which is used during the problem solving process and is intrinsically linked to the construction of the mental model. The definition of gestures varies in the literature. For instance, Roth (2001) defined gestures as hand movements made with a specific form where "the hand(s) begin at rest, moves away from the position to create a movement, and then returns to rest"(as cited in Rasmussen, Stephan, & Allen, 2004). Although there are many definitions present today, we define gestures similarly to Roth: gestures are hand movement(s) where the hand(s) extends in an outward position, makes a movement or movements consisting of icons, symbols, and indices, and then returns to its normal position. Icons include gestures that demonstrate a thing, symbols are those that describe a thing, and indices are gestures indicating a thing (Clark, 1996 as cited in Rasmussen, Stephan, & Allen, 2004).

There are two types of gestures we identified: dynamic and static gestures. Dynamic gestures consist of moving the hands to describe the action that occurs in the problem or movements made to represent mathematical concepts. Within dynamic gestures there are two subcategories: dynamic gestures related to the problem (DRP) and gestures that are not related to the problem (DNRP). Static gestures are done to illustrate a fixed value (length, constant radius, etc.) or to illustrate a geometric object. Static gestures consist of static gestures related to a fixed value (SRF) and gestures related to the shape of an object (SRS).

Dynamic Gestures

We define DRP as hand gestures that consist of movements describing parts of the students' diagrams, whether it is the motion of an object or changing rates/values. DRP is further broken down into two subcategories. The first subcategory identifies hand gestures used to answer/clarify a concept/question to a classmate. A student may use a hand gesture to reason the problem out. For instance, a group of students, Jackie, Josh, and Cathy, were trying to solve the following related rates problem (the boat problem):

> A boat is pulled onto a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is 1 m higher than the bow of the boat. If the rope is pulled in at a rate of 1 m/s, how fast is the boat approaching the dock when it is 8m from the dock? (Stewart, 2009, p. 132)

Although the problem in the textbook has an image depicting the situation, students were not provided with the image. They had to construct the diagram and solve the problem.

Josh: You know what a pulley is right?
Jackie & Cathy: No
Josh: It's a little thingy [HG*: raises his right hand in the air, rotates his right index finger inward, then raises left hand to the same height as his right, puts both hands together, and pulls downward*] you pull…so it's gotta be on top.
Jackie & Cathy: Ohh….
Jackie: So it's going to be like that?
Josh: So… [*Drawing diagram*]
Cathy: I did it like that [*laughs a little*]
Jackie: See, how do they expect us to not know what a bow is
Cathy: Yeah…
Josh: You don't know what a boat is?
Jackie: I know what a boat is…
Cathy: But not a bow…or whatever

The gesture made in this clip is also characterized as a symbol because Josh describes a pulley with his hands. Although his peers, who did not know what a pulley was, prompted Josh's gesture, the gesture revealed his mental representation of the problem. After he makes the gesture, he begins drawing his diagram and concludes that the pulley must be on top. The gesture influenced Josh's perception of the problem (i.e. placement of the pulley), which also had a role in how he labeled his diagram. After this exchange, Jackie constructs an appropriate mental model of the situation as is evidenced by her subsequent exchange with Cathy.

The second subcategory deals with hand gestures done in order to understand and reason about the problem. For instance, Jackie is trying to solve the boat problem by first attempting to understand the dynamic element of the problem.

Jackie: [HG*: Jackie moves her hands in a circular inward motion (Figure 1)*] Is the…when its being pulled, it's being pulled from there?
Cathy: Yeah
Jackie: [HG*: Jackie points her left index finger towards Cathy's diagram (Figure 2)*] so then it's approaching on *x*, so okay we are looking for *dx/dt*, but then we're looking for this rate…

Figure 1: Jackie's gesture for boat

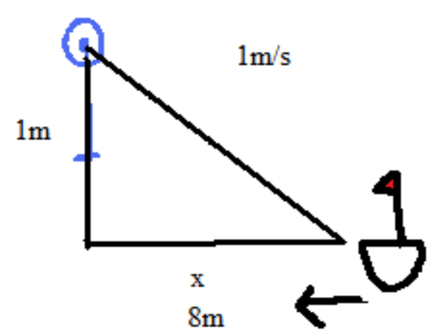


Figure 2: Cathy's drawn diagram for problem

Jackie's hand gesture is prompted because she is trying to understand the problem both conceptually and physically. In the first hand gesture, she is portraying the pulley with the use of her hands, which Clark (1996) classified as a symbol, because she is describing the pulley with her hands (as cited in Rasmussen,

Stephan, & Allen, 2004). With the second hand gesture, she references Cathy's diagram (Figure 2) to understand what is happening to the boat. Here, Jackie is utilizing an index gesture, because she is indicating Cathy's diagram to observe the behavior of x as the pulley is pulled. Through the first and second hand gestures, she sees that as the rope is pulled, the boat moves closer to the dock. She recognizes that the change is occurring on x, and hence, she relates the change in x with dx/dt. The gesture made here, appropriated in part from Josh's earlier gesture, allows her to conceptualize the diagram described in the problem. The gesture acts alongside Jackie's constructive thinking as she tries to comprehend the situation at hand.

For DNRP, students and/or SI leaders use hand gestures to refer to mathematical concepts. The SI leader assisted Susan, Brian, and Cesar with the trough problem which states:

> A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of 12 ft^3/min, how fast is the water level rising when the water is 6 inches deep? (Stewart, 2009, p.132)

*SI Leader***:** Now, what do you do to find what… cause you're trying to find *dh/dt* [HG*: He first moves his right hand right to left, with his fingers curved in the shape of a c, and then he changes the position of his hand and moves it up and down*] right?
*Susan***:** Yeah. So we have to take the derivative of each side.
*SI Leader***:** and that is when you plug in what your paused… Do you see it Cesar?

When the SI leader makes the hand gesture, his motion depicts the fractional aspect of the derivative (i.e., *dh* over *dt*). Additionally, Susan seems to associate finding *dh*/*dt* with implicit differentiation because she immediately thinks about taking the derivative of both sides when the SI leader mentions *dh*/*dt*. In another clip, Jackie explains to Cathy the difference between taking the derivative in a related rates problem, and taking the derivative of a (usual) function of x.

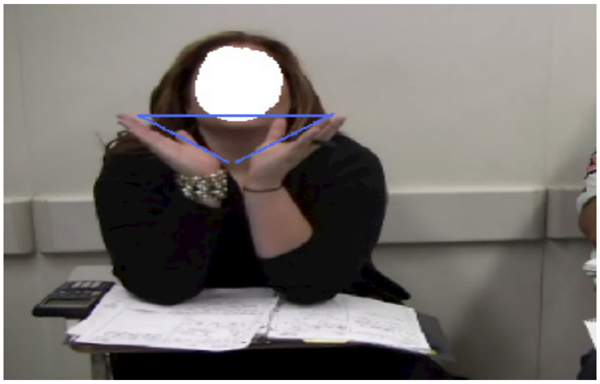
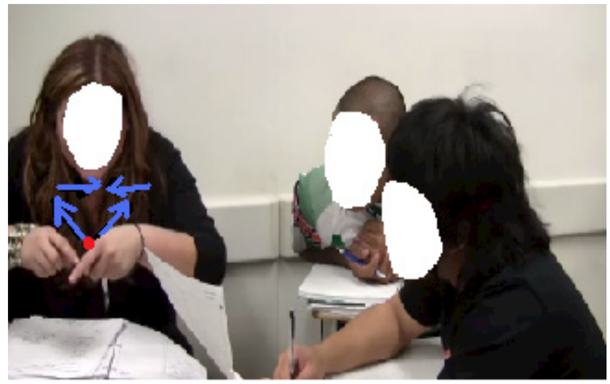
Cathy: Jackie I have a question [*laughs*]. Remember how last time you said we always keep *dy/dt*
Jackie: Mhmm.
Cathy: Do we also keep *dx/dt* cause it's not the same?
Jackie: Yeah…no, because we're not solving for uhmm… [HG*: lifts right hand up then moves it downward in a diagonal manner*] it's no longer like a just taking the straight out derivative of it, cause we have different properties that we need to relate together.

The hand gesture was a symbolic representation of a derivative. Along with her hand gesture and her explanation to Cathy, it can be seen that Jackie is able to discriminate between the derivatives in the context of a related rates problem and generally taking the derivative of some function of x. Although she does not explicitly state that the difference is based on taking the derivatives with respect to time, that underlying concept somehow triggered Jackie to separate the two. The gesture, in this case, did not act as part of Jackie's constructive thinking about the problem; rather the gesture was used as an explanatory aid.

Static Gestures

Apart from dynamic gestures, we identified static gestures, which consist primarily of gestures that illustrate a geometric object or refer to a fixed value (length of one the sides, constant radius, etc).We will distinguish them as static related to the shape of an object (SRS) and static related to a fixed value (SRF). SRS is focused on students or SI leaders utilizing their hands to depict the geometric shape of an object; we also consider referencing the general diagram as SRS. Here we see Susan trying to process the trough problem. Figure 3 shows Susan's initial image of the trough problem. She traces with her fingers the outline of tip up standard equilateral triangle. As seen in Figure 4, however, Susan changes the orientation of the triangle to a downward position as she reasons out the scenario in a realistic setting. Although not much is said, Susan's facial expression and gestures illustrate her thought process of trying to understand the general shape of the trough.

Susan: [HG*: elbows are bent and on top of desk, wrists are touching, hands are open diagonally, and pointing in opposite directions, flickers her left hand as she moves her pencil between her fingers, puts hands together, then pulls them apart in a diagonal direction Figure 4)*] so it's not the sides…it's the width [*SI Leader interjects and provides insight on the problem*]

Figures 3 & 4: Susan gestures to describe the trough

Her first gesture illustrates a cross-section of the ends of the trough; as she continues to think, she forms the length of the trough by pulling apart her two hands. Susan, along with many students, struggled to understand the geometric aspect of related rates problems. Engelke (2005, 2007) indicated that students tend to adopt a procedural way of thinking when they approach related rate problems. This may be why students have a hard time understanding problems such as the trough problem, which deal with a three-dimensional object, as well as applying the concept of similar triangles. For instance in the trough problem, Susan automatically drew an upright equilateral triangle (Figure 3) as opposed to visualizing the triangle upside down or oriented in a different way. Students with geometric misconceptions tend to construct incorrect diagrams, leading them to the wrong solution. If the SI leader had not intervened, the students would have attempted to solve the problem with an incorrect diagram, hence leading them to the wrong answer. That said, we consider SI leader intervention is necessary at times to provide students with guidance on challenging problems. The SI leader's help can start the students on the path to solving the problem correctly, without just giving them the answer. However, sometimes intervention by SI leaders, as also revealed in Scherr (2008), may actually interrupt a student's thinking.

The discussion below provides a glimpse as to how students think about a challenging problem.

Susan: Okay so like the square one is ten feet [HG: hands are both raised, fingers spread out flat, she then moves them down, putting hands in a diagonal position] like if you look at this from the top

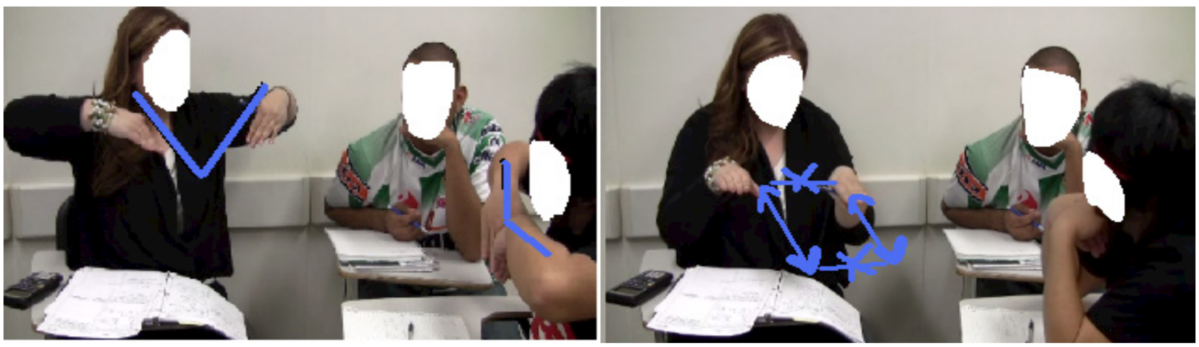
Brian: It's like this right? [*HG*: *mimics Susan's gesture (Figure 5)*]

Susan: Yeah, the sides are cause you have the square part [*HG: hands are moved front to back then she brings them together, to denote the shape of a square (Figure 6)*] and that's the base [*HG: moves her left hand, which is extended and flat back and forth in a vigorous manner*], which is ten, but [*HG: she lifts her hands, elbows bent, hands in a diagonal manner, with hands in this position, she moves both index fingers back and forth*] then you have the three feet wide triangles that are coming down into it [*HG: moves hands that are elevated and in a diagonal position, down , closer to each other*], so [*HG: hands laid flat, palms facing the ground, she moves her hands left to right*] that's why it's not just a flat, it's not like a square box [*HG: moves both hands vertically*], it has that extra [*HG: elbows bent and tilted, hands are lifted up in a diagonal manner, then moves them a bit towards each other*] side coming in… so from if you look at it kind of from the top [*HG: hands are slightly up, moved up and down swiftly*] its coming down like that [*diagramming, then shows it to group members*] like that's what the inside looks like

Brian: Yeah

Susan: …if you are looking at it and this is your bottom length, that right there, which is ten, [*HG: hands flat, lifted from desk, she moves them swiftly together and apart, then lifts them up*] so you can't just say its ten on the top and bottom, which is like what we were doing, on, in class…

Although Susan's mental picture of the diagram itself is not correct, her gestures show that she is engaged in constructively thinking about the solution to the problem. She gestured to describe the picture she was visualizing in her mind. She was able to transfer that mental image into a more realistic perspective with the use of gestures. Since she visualizes the diagram with a square bottom, she reasons that the top and bottom could not be the same because they have different lengths. She realized that the trough cannot be a square box, because the sides are diagonally positioned. She also seemed to recognize similarity between this problem and those discussed in class. The gesture that Susan made also prompted one of her group members to engage in the problem solving process. When she made the first gesture, Brian mimicked the same gesture (Figure 5), which showed that he was also thinking about the problem in an abstract manner.



Figures 5 & 6: Susan engages in describing the shape of the trough with her hands

For SRF we take into consideration the students' gestures done with respect to a fixed value, such as length or constant radius. These gestures are usually associated with students' diagrams. As mentioned above, SRF deals mostly with students referencing a fixed value. In this session, Brad and Mark work on a related rates problem that deals with the distance between two cars moving in different directions.

Brad: [*diagramming*] Specific, its saying, one is traveling south at 60 miles per hour
Mark: [*mumbles something*]
Brad: I don't know. So you times it, so 2 times 60 [*labels*], cause the two hours, [HG: *moves his left hand top to bottom*] that's the length, and 25 is the top.

Brad's gesture was done in order to describe a fixed value, in this case, length. Although his gesture was quick, it was done in an effort to explain to Mark the values corresponding to their specific diagram. This type of gesture shows how students associate given values to their diagrams.

Conclusion

A strong relationship between diagramming and the two types of gestures identified in the study is evident. Static and dynamic gestures were often used in regards to the students' diagrams, but static gestures seem to have a stronger relationship to diagramming as they deal with the diagram itself. In order for the student to even attempt solving the problem he or she began with drawing a diagram. It appears that when the students were stuck with their diagram or parts of their diagram, they gestured while trying to reason out the part about which they were confused. Several students gestured because they were trying to obtain a better understanding of how the diagram corresponded to geometric terms in the given problem. The more challenging the problems were, the more the students gestured. Some gestures were influenced by prior gestures and students quickly adopted and adapted gestures made by their peers. For instance, in the boat problem with Jackie, Josh, and Cathy, before Jackie made the gesture to describe how the boat

Van Zoest, L. R., Lo, J.-J., & Kratky, J. L. (Eds.). (2012). *Proceedings of the 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Kalamazoo, MI: Western Michigan University.

was being pulled by the pulley, Josh's gesture, which described the pulley, had to occur first. Other times gestures arose because there was a lack of knowledge that was needed to even begin the problem. An example would be the trough problem that Susan's group was assigned. Her gestures were caused partly because neither she nor any of her peers knew what a trough was. In her mind she pictured an object with a square bottom and triangular sides. Since she did not know what a trough was, most of her gestures were done to figure out what the trough looked like. Most gestures have one thing in common; they were made to solve the problem by first understanding the problem abstractly. Although not much research has been done on the impact that gesturing has on undergraduate mathematics students, one must wonder whether or not the gesturing that occurs is beneficial to students. There is no denying that gesturing does influence the way students approach a problem, but to what extent?

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